Optimal divestiture strategies under the threat of hostile takeover

A game theoretic model

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1 Introduction

In recent years, corporations have recognized the potential of unlocking value by carefully planning their divestment strategies and refocusing on core business. This phenomenon is well documented in a 2016 survey done by Ernest&Young\(^1\). They look at more than 900 corporations, each of which had already divested in previous years, and report that almost half of the companies are expected to divest again by 2018. The main types of divestiture strategies used today are sell-offs, carve-outs, spin-offs, and tracking stocks. Although, these strategies have different features, implementations and motives, they all aim at providing value to shareholders and the parent company. While M&As have been intensively studied in the last few decades, corporate divestitures started gaining attention only after the 1980s, and since then represent popular restructuring strategies worldwide. For example, the number of divestitures in 2015 alone amounts around 13,000 with a volume of around 400 billion dollars, which corresponded to almost 39 percent of worldwide restructuring transactions of the same year.\(^2\)

McKinsey (2001) reports a particular phenomenon regarding equity carve-outs which is at the base of the original model that we construct in this paper. In a study conducted by analyzing more than 200 carve-outs they find that in 92% of the cases the ownership over the subsidiary falls below 50% over a five years time frame (i.e the parent company does not retain full-control over the carved-out unit). Even more impressive, the article reports that 31% of the parent companies end up retaining less than 25% of their subsidiary in the same period, therefore increasing the probability that in the case of a hostile takeover bid the parent company could not control the outcome. What McKinsey ultimately wants to show regarding carve-outs is that most of them “... do not create shareholder value unless the parent company follows a plan to subsequently fully separate the carved-out subsidiary.”

In this paper we explain this insight by proposing an original model to analyze the divestiture strategies of a firm which in a first period faces the decision of whether

\(^1\) EY(2016), Corporate Divestment Study
to carve-out a subsidiary or not. In case of a carve-out, the firm subsequently decides to either fully separate the subsidiary through a sell-off, or to try to retain a majority stake. The novel feature of our model is that we account for the possibility of a hostile takeover of the subsidiary by a competitor whenever the firm decides to carve-out in the first stage. This is because of the reduced control over the business unit and a possible further dilution of shares. Takeover threat is crucial to our model in the sense that it shapes the competitive framework in which Firm 1, the divesting company, competes. As we will explain in depth throughout the paper, by obtaining the subsidiary, rival companies can enjoy some synergies and reduce marginal costs, leading to different Cournot equilibria and therefore profits of the divesting firm in case of a hostile takeover. Finally, as an extension to our baseline model, we will also include the possibility that after carving out there is positive probability that there are no non-competitors to sell the subsidiary to. Therefore, in this case the firm can only sell to a direct competitor after the carve out or try to fight off a hostile takeover. We then analyze how this affects the firms decision to divest in the first stage.

In the baseline model we assume that the firm has already carved out the business unit and now faces the decision of whether to sell-off voluntarily to a non-competitor or to try to retain the control and face the threat of a hostile takeover. We show that in this environment if the probability of a hostile takeover is high, then the optimal strategy is consistent with McKinsey’s findings, namely the divesting firm should directly sell-off to a non-competitor. Conversely, if the probability of takeover is low, then the divesting firm shouldn’t sell.

After analyzing the baseline model we extend it by accounting for the probability of not having a non-competitor willing to buy the carved-out subsidiary, which together with the probability of takeover, will shape three different optimal divestment strategies. When the probability of takeover is very high, the divesting firm should initially carve-out if and only if there’s a high chance of finding a non-competitor willing to buy the subsidiary. If instead there is a low chance of finding a non-competitor willing to buy the subsidiary and the takeover threat is very high, then the divesting firm should not carve-out in the first place. If instead, the probability of a hostile
takeover to occur is moderate (i.e. not too high and not too low) then the firm will always carve-out initially, then in the subsequent period it will sell the subsidiary if there is a non-competitor or try retain the ownership if there is no non-competitor willing to acquire it. Finally, like in the baseline model when the probability of hostile takeover is very low the optimal strategy for the divesting firm is to carve-out and to try to retain ownership.

The rest of the paper is organized as follows. Section 2 will provide a brief review on restructuring and more specifically on divestiture strategies. Section 3 reports an overview of the literature on carve-outs and sell-offs. Section 4 presents the baseline model and the extension to it, together with the results of the paper. Section 6 provides a summary of the main findings.

2 Review of Divestitures

2.1 Corporate Restructuring

Corporate restructuring is the process of changing the company’s financial structure, management and business model in order to maximize the efficiency and the profitability of the firm. In other words corporate restructuring involves all of the types of operations aimed at increasing stakeholders’ value from the company. Corporate restructuring strategies usually include mergers and acquisitions (M&A) and corporate divestitures. In the set of restructuring strategies we also find financial restructuring processes such as leveraged buyouts (LBO)\(^3\), management buyouts (MBO)\(^4\) and recapitalization.

In the restructuring decisions, companies can have two different approaches: a reactive behavior or a proactive behavior. A reactive behavior is shown whenever the firm tries to find a remedy to previous acquisitions or business activity. Usually when the outcome of a strategic corporate decision is below expectation firms can decide to

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\(^3\)When a group of private investors purchases all of the equity of a public corporation and finances the purchase primarily with debt (Berk, DeMarzo and Harford (2012))

\(^4\)A leveraged buy-out in which the buyer group includes the firm’s own management (Berk, DeMarzo and Harford (2012))
pursue some restructuring operations, for example a reduction in the diversification of
the business with restructuring techniques meant at refocusing on the core business.
Other common restructuring transactions following this behavior aim at reacting to
competitive pressure in a specific sector, eliminating negative synergies and solving
governance problems.

A proactive behavior instead, implies the decisions of a company to divest or
transfer part of the business in order to pursue new market opportunities. Corporate
restructuring strategies can be described according to three different dimensions:
portfolio restructuring, financial restructuring and operational restructuring. Financial
restructuring strategies involve changes in the capital structure of the firm. These
types of operations are usually undertaken when a company is either over-leveraged
or under-leveraged. In the former case restructuring can be implemented for example
through issuing new stocks, re-negotiating the debt and issuing debt-to-equity swaps
\(^5\), while in the latter case, common transactions include LBOs, borrowing funds or re-
purchasing shares. Operational restructuring aims at improving corporate efficiency
through changes in the organizational structure such as internal re-organization and
downsizing of the labor force. The last category, is the one on which this paper will
be focused, namely portfolio restructuring. These types of operations are usually
carried out through M&As and divestitures, and they are intended to strategically
optimize the structure of the company.

\subsection{Corporate Divestitures}

In a broad sense, corporate divestitures are those portfolio restructuring practices
involving the sale of stocks or of a part of the business of a company. These types
of operations are aimed at improving the company’s value and enhancing growth
by refocusing and restructuring internal lines of business. When talking about di-
vestitures it is necessary to distinguish two types of operations in which the divested
item differs. The first is assets’ divestiture, where the company divests, partially or

\(^5\)Transactions giving the borrowers the right to transform debt claims, such as loans, into shares
of the creditor company.
fully, physical assets and organizational resources, where the second type is business divestiture. Business divestiture includes transactions of business divisions, subsidiaries or units, and implies a change in the portfolio holdings of the company. When analyzing the divestiture strategy, a company must be able to first recognize the drivers of the decision, namely the problems the company faces, and subsequently set goals to be reached. In this strategic evaluation, the main decision the managers have to take is the choice of corporate divestiture strategy. Further, the object of the divestiture can vary according to the situation and similarly the types of strategies differ from one to the other in terms of strategic fit to solve a specific problem. There are four main types of corporate divestitures, namely: sell-offs, carve-outs, spin-offs, and tracking stocks.

2.2.1 Sell-off

A corporate sell-off is a divestiture operation in which a parent firm sells some of the assets in its portfolio, which can include both physical assets and business subsidiaries. Through sell-offs the parent firm fully relinquishes its ownership and control over the business unit, implicitly increasing the focus on the part of the company that has not been divested. Usually sell-offs are private transactions between the company and a direct seller, which is typically another firm or a private equity fund. When selling assets the parent company receives cash or shares depending on the nature of the disinvestment decision. The proceeds gained from the transaction then have two different methods of dispersion; in some cases they are distributed to the shareholders or to creditors in order to repay debts or provide liquidity, in other circumstances the earnings are retained and used to fund other investment opportunities. After a sell-off transaction both the ownership and the management of the company’s subsidiary shift to the acquirer but the legal entity of the parent firm remains unchanged and there is not the creation of a new firm, differently from what happens with other forms of divestiture strategies such as spin-offs and carve-outs.
2.2.2 Equity Carve-out

Equity carve-out (ECO) is the partial initial public offering (IPO) of a company’s subsidiary; the sale of part of a business unit in the public market. The shares issued during the IPO can be directly sold by the subsidiary in a primary issue and by the parent in a secondary issue. In both cases the parent firm decides the amount of shares available to outside investors. The proceeds from this transaction depend on the stake that the company decides to sell to the public and on the market evaluation of the subsidiary. After the carve-out the proceeds from the IPO can either be left in the subsidiary or can be transferred to the parent company to finance the whole conglomerate. As in the case of sell-off, the ownership structure of the company changes and there is the possibility of a change in the managerial body, but differently from the previously explained form of divestiture, ECOs imply the creation of a new type of business from a legal point of view. A factor of great importance in the analysis of this type of restructuring strategy is that equity carve-outs are tax exempt. In fact, if the company sells newly issued shares there is no tax burden both for the company and for the shareholders. ECOs are often used as transitory strategies towards different types of divestitures and a key driver behind this phenomena is that companies can partially reduce the taxation on the divestiture transactions by undergoing an equity carve-out.

2.2.3 Spin-off

Spin-offs are types of divestitures that don’t provide any cash inflow to the parent company, in fact this type of portfolio restructuring strategy consists in the transfer of the stock of a subsidiary to the already existing shareholders of the company. By doing so a new corporation is developed and the parent loses the control over the asset together with its ownership. As a variant of spin-offs we can include split-off transactions, where the parent company decides to distribute the shares to shareholders in exchange for some of the parent firm shares.
2.2.4 Tracking stocks

Tracking stock is a relatively new form of restructuring compared to the three above-mentioned. These are special type of stocks issued by the parent company for one or more of its subsidiaries that are directly tied to the performance of the specific subsidiary. Tracking stocks therefore allows the shareholders to invest only in a part of the company and gives the managers the ability to retain control while raising new capital.

3 Literature Review

Studies on corporate divestitures gained increased popularity during the 1980s, a period which was a turning point in the analysis of corporate strategies. In the 80’s, after the introduction of new anti-trust regulations and the bad performance of previous acquisitions, there was a tendency toward reversing the diversification strategies widely undertaken during the 1960s through divestitures. Ravenscraft and Scherer (1987) showed that from the 1960s to the 1970s almost 33% of the acquired companies were later divested. Kaplan and Weisbach (1992) later estimated that 44% of acquisitions from 1971 to 1989 were follow by some form of divestiture.

Literature on carve-outs and sell-offs includes mainly empirical papers analyzing wealth effects of these divestitures on the companies and shareholders or studies regarding the motives behind carve-outs and sell-offs. Few papers relate them together and the majority of those are empirical studies do not focus on the strategic decisions that may bring a corporation to sell-off a business unit after having carved it out.

Schipper and Smith (1986) find positive price reactions to equity carve-out announcements and they comprehensively report the different motives for a company to deploy this restructuring strategy. Apart from financing reasons they consider ECO as a way to reduce asymmetric information; through the publication of financial statements the parent company reduces private information between the managers and investors and lets the market reveal the hidden value of the subsidiary, which is usually more difficult to uncover as a part of the whole company.(see Powers, E. A.
This also poses the base to discard the negative conception of divestitures as a way of divesting an under-performing business unit. Nanda (1991) outlines how the decision to carve-out doesn’t automatically signal that a subsidiary has a lower value, reasoning that rational managers usually decide to let a firm be monitored by capital markets when the underlying assets are considered to be valuable. Holmström and Tirole (1993) showed how markets can act as a monitor of managerial performance. Another strategic reason for ECO is related with managerial incentives. For example, Schipper and Smith (1986) argue that issuing shares to restructure a company’s portfolio can lead to a reduction in the agency costs due to misaligned incentives of managers and shareholders. Namely, since in the majority of the cases after a carve-out, contracts are modified in order to tie managerial compensation with stock prices, this poses an incentive for managers to take decisions that enhance share value, thus aligning the interests of shareholders and managers. Finally, equity carve-out doesn’t always represent a final stage in divestiture strategies, instead this type of restructuring is often followed by a subsequent action. As mentioned above, McKinsey (2001) suggests that it is optimal to undertake an equity carve-out only as a temporary step in fully divesting the subsidiary.

Reasons for sell-offs are mostly similar in the fact that they represent a way to finance new operations or repay debts. John and Ofek (1995) show that sell-offs can enhance shareholders’ wealth by increasing the firm’s focus on core business. Additionally sell-offs can be deployed as a defense measure for hostile takeovers through widely known kamikaze strategies such as the sale of crown jewels or the scorched Earth policy which involve the sale of assets in order to make the company less appealing to bidders. Bhagat, Shleifer, and Vishny (1990) showed that when the threat of a hostile takeover arises, as it is in our model when the parent firm retains the already carved-out subsidiary, the assets are sold at a lower value than they

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6A takeover defense tactic that involves the sale of the target company’s prized and most coveted assets - the "crown jewels" - so as to reduce its attractiveness to the hostile bidder. (Auerbach, Alan J.; Richard S Ruback (1987) "3 An Overview of Takeover Defenses").

7A takeover prevention strategy in which the target company seeks to make itself less attractive to hostile bidders by selling off assets, taking on high levels of debt or initiating other activities that may damage the company if it is purchased (Wall Street Words: An A to Z Guide to Investment Terms for Today’s Investor by David L. Scott).
would be otherwise. Moreover, the probability of facing a takeover threat improves internal decision making, influencing the decision of the company to divest.

Similar to our research, Zingales (1995) models a game in which an owner who wishes to sell his firm decides whether to go public, through an IPO, before selling to potentially obtain a higher price. If he issues an IPO, a rival arrives and makes a tender offer and, by assuming that the offer succeeds with probability one, the company is sold. A crucial assumption in his model is that the whole company is being sold to a rival whose evaluation of the company is always higher than the one of the incumbent. In this way Zingales doesn’t consider the competition effects which instead arise when only a part of the business is sold. In our model, differently from his paper, we do not assume that the firm wants to sell ex-ante but rather that this decision is based on the effects of owning a majority share in the carved out business (which yields cost reducing synergies with the parent and therefore effects the competitive environment). If the selling firm instead decides to retain after carving out, it faces the threat of a hostile takeover by a direct competitor. We show that when the probability of hostile take over is large enough, the firm always finds it optimal to sell-off after the initial carve-out.

Zingales shows that whenever the buyer values the firm more than the incumbent, the acquired firm can gain more by first carving-out and then selling subsequently to the rival. The main forces driving these results are that the incumbent can free ride off of the value added to the firm by the rival during the bargaining process, thereby extracting surplus. In contrast, we extend our model to include the decision of whether to carve-out or not when there is some probability that there are no non-direct competitors who value the firm high enough to make an offer (before or after the carve-out). We show that this exogenous probability is only relevant to the firm’s divestiture decisions when the probability of hostile take over is large. In this case, it is optimal to carve-out and subsequently sell only when the probability that there is a buyer who is not a direct competitor is high enough. In contrast, if the probability of finding a buyer who is not a direct competitor is low enough, it is always optimal to not carve-out the firm, nor sell privately. These insights illustrate the effect of competition on the decisions of the firm to carve-out/sell-off a subsidiary.
4 The Model

4.1 Economic setting and specification of the model

We consider a model with two firms, Firm 1 and Firm 2, who compete in quantities à la Cournot. To start, we assume that Firm 1 has recently carved out one of its subsidiaries which faces the threat of a hostile takeover by Firm 2. This phenomena is common among businesses in the need of further funding for operations and investments. When the company carves out, it sells a stake in the ownership of one of its divisions or subsidiaries to outside investors in order to finance internal and external projects. As described also in McKinsey (2001), even if all these factors foster growth, they lead unequivocally to a dilution of the parent’s holdings over the subsidiary, making it easier for an outsider to obtain the control over a company’s carved-out division. Another factor which may lead to an increased probability of hostile takeover is that, after carving out a part of the business, an outside raider can bid directly for that part of the company rather than having to bid for the parent firm as a whole. This could be profitable to the outside acquirer in the case that only the carved-out division is in line with its core businesses or if the manager of the bidding company has better know-how in running the divested branch than the current one (Chemmanur and Paeglis (2001)).

The two competing firms have specific characteristics based on the ownership of the resulting carved out business. We assume that both Firms 1 and 2 are diversified (see Steiner (1997)), in the sense that their different lines of business are uncorrelated with each other and require different management capabilities to be properly run. The value of the subsidiary to either firm is in its ability to lower the parent firms marginal cost. This is because, by diversifying, a company can exploit shared resources and capabilities in diverse business activities. The main cost reduction drivers that arise from diversification are cost synergies and economies of scope. By definition, economies of scope exist when using a resource across multiple activities uses less of that resource than when the activities are carried out independently (see e.g. Collins (2001)) which allows companies to reduce marginal costs. More-
over, through synergies the company exploits common strengths, derived from the combination of different business activities, reducing marginal expenditures. Taking this into consideration, we assume in our model that if one of the two competing firms owns a majority share in the subsidiary then this leads to a reduction in their marginal cost from $\alpha > 0$ to $\hat{\alpha} < \alpha$.

Prior to competing in the market, Firm 1 can make a choice of whether to sell the subsidiary to a non-competitor, which we denote by action $S$, or to retain ($R$) the firm and face the risk of hostile take over. We assume the probability that Firm 2 takes over the subsidiary is $\beta$, otherwise with probability $1 - \beta$ Firm 1 retains ownership of the subsidiary. In this model we take $\beta$ as an exogenous variable expecting it to tend toward 1 in the long run. This increase in the probability of takeover is due to possible conflicts of interests between shareholders of the parent firm and shareholders of the subsidiary. What we mean by this is that in any subsequent period, the subsidiary may need further capital to fund new profitable projects but the parent may prefer to use available resources in other ways. In this scenario, in order to finance positive net present value projects available to the subsidiary, the subsidiary may decide to issue new shares. This solution provides the subsidiary and parent firm with a cheaper ways of obtaining funds, but reduces the parent’s ownership of the subsidiary by diluting its shares. As reported in McKinsey (2001), of the 200 companies they considered for their study, after a carve-out only 8% retain the ownership of the subsidiary while nearly 40% are ultimately acquired by third parties. A specific example reported in the McKinsey study exactly represents this phenomena. In 2001 Siemens decided to carve-out Infineon Technologies, one of its subsidiaries, selling almost 30% of the ownership in the partial IPO. By the end of 2001, because of funding issues, the parent decided to reduce its the stake over the divested subsidiary from 70% to almost 50%, subsequently further reducing its stake until 2006 when Infineon was fully divested.

Once Firm 1 has decided whether to sell or try to retain the subsidiary, both firms compete in quantities $q^k_i$ where $i \in \{1, 2\}$ and $k \in \{S, T, R\}$. Namely, $q^k_i$ is the quantity set by Firm $i$ given the competition environment $k = S, T, R$; sell to non-competitor, take over by competitor, or retain ownership, respectively. We
assume that the demand for the product is given by the linear function \( p(q^k_1, q^k_2) = A - b(q^k_1 + q^k_2) \). Further, as a normalization we assume that \( \alpha = 0 \). Therefore, the profits of Firm \( i \) in environment \( k \) is

\[
\pi^k_i(q^k_1, q^k_2) = p(q^k_1, q^k_2)q^k_i - 1(\text{Don’t Own}) \cdot \alpha q^k_i
\]

where \( 1(\text{Don’t Own}) \) is 1 if firm \( i \) does not own the subsidiary and 0 otherwise. When Firm 1 looses the ownership over the subsidiary, either by selling it voluntarily or because of the hostile takeover, it also gets an amount \( S \) which represents the price at which the subsidiary is sold. Consequently in the model we will refer to \( \Pi^k \) as the profit in environment \( k \) including the selling price of the subsidiary, while to \( \pi^k \) as the profit in environment \( k \) without including the selling price.

### 4.2 Cournot Equilibria

In the calculation of the Cournot equilibrium for each of the cases specified above we define as \( \alpha q_i \) the cost of firm \( i \) whenever it does not own the subsidiary. When instead one of the two firms has the ownership of the division, its cost is represented by \( \hat{\alpha} q_i = 0 \). Where the reason \( \hat{\alpha} \) is lower than \( \alpha \) is because, through owning the subsidiary, the parent firm either reduces expenses or increases productivity. The difference between \( \alpha \) and \( \hat{\alpha} \) can then be interpreted as a the loss in positive synergy between the parent company and the subsidiary or the reduction in economies of scale. We assume that without ownership over the subsidiary the two competing firms are identical, so that when the parent company decides to sell \( (S) \) to a non-competitor neither Firm 1 or Firm 2 will have the synergy. For this reason, we assume that, in this case, the costs of Firm 1 are identical to the costs of Firm 2 not changing much predictions of our model.

In this scenario through Cournot competition both Firm 1 and Firm 2 produce the same quantity in equilibrium with the relative price, that is (see the appendix for all
considering the cost function as specified previously, the profits of the parent company when, after the carve-out, it voluntarily decides to sell off the subsidiary to a non-competitor are:

\[ \Pi^S = \frac{(A - \alpha)^2}{9b} + S \]  
(1)

where \( S \) is the price at which the third party buys the division of Firm 1. When Firm 1 decides instead not to sell, the firm faces the takeover threat because of the reduced ownership in the subsidiary due to the carve-out. Therefore, it can be possible that the subsidiary will be acquired against managerial will, at a price \( S^T \) lower than price \( S \) in equation (1), at which it would have been sold if Firm 1 divested voluntarily. Given that this doesn’t change the result much we will not account for this difference, therefore we assume \( S^T = S \). The equilibrium quantities, price and profit are then computed considering \( \hat{\alpha}q_i = 0 \) as the costs for Firm 2, since it now enjoys the positive synergy, and \( \alpha \) the costs of Firm 1. Competing à la Cournot, in the takeover environment (T), Firm 1 and Firm 2 produce respectively:

\[ q_1^T = \frac{(A - 2\alpha)}{3b} \quad q_2^T = \frac{(A + \alpha)}{3b} \]

with price being:

\[ p^T = \frac{(A + \alpha)}{3} \]

the profit of Firm 1 in equilibrium will therefore be:

\[ \Pi^T = \frac{(A - \alpha)^2 - 2\alpha A}{9b} + S \]  
(2)

if Firm 1 instead manages to retain the subsidiary it will take advantage of having the synergy and it will produce more than Firm 2, the quantities for the two firms
Figure 1: Baseline model

will be reversed:

\[ q_1^R = \frac{(A + \alpha)}{3b} \quad q_2^R = \frac{(A - 2\alpha)}{3b} \]

with the same price of the previous case, namely:

\[ p^R = \frac{(A + \alpha)}{3} \]

and the profit of Firm 1 is equal to the revenue, (since we assumed \( \hat{\alpha} \) to be equal to 0) i.e.:

\[ \Pi^R = \frac{(A + \alpha)^2}{9b} \]

(3)
4.3 Analysis of the model

In order to present the results of the Cournot equilibria we first need to consider what $S$ is and give a measure for its value. As previously presented $S$ is the price at which the non-competitor buys the subsidiary. If we call $\bar{S}$ the most the non-competitor is willing to pay and consider that the least the seller is willing to accept is $\Pi^R - \Pi^T$ (See appendix.), then we expect $\Pi^R - \Pi^T \leq S \leq \bar{S}$. In order to make the problem interesting we assume that this value is such that $\Pi^R$ is greater than $\Pi^S$. Otherwise, the firm will always prefer to sell after a carve out which it is not always the case in reality. Having said this, we can draw some findings from the equilibria.

**Lemma 1.** According to the Cournot equilibria, the profit from retaining the firm, $\Pi^R$, is larger than that of when the subsidiary is taken over, $\Pi^T$.

*Proof.* See Appendix.

Therefore, disregarding the possibility of takeover, Firm 1 would always be better off by retaining. However, as specified in the previous section, when Firm 1’s decision is to retain the firm, with probability $1 - \beta$ the company will manage to retain the ownership over the subsidiary and with probability $\beta$ it will face a hostile takeover from a direct competitor. It follows that the expected profit of Firm 1 when it does not sell to a non-competitor will be $\beta \Pi^T + (1 - \beta) \Pi^R$.

Therefore, if the probability of takeover is equal to 1, Firm 1 is better off by selling the subsidiary directly to a non-competitor. On the contrary if, in the same scenario, the threat of takeover did not exist, the parent company would prefer to retain the ownership. We will now characterize the equilibrium of the game in Figure 1.

**Proposition 1.** There exists $\hat{\beta} \in (0, 1)$, such that whenever the probability of hostile take over is greater than $\hat{\beta}$ the firm prefers to sell to a non-competitor. Otherwise, the firm prefers to attempt to retain the subsidiary and face the threat of hostile

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*Namely, as we show in the appendix, this implies that $S < \frac{4\alpha A}{9\beta}$.***
takeover. By setting the profits of selling to a non-competitor equal to the expected profit of not selling, we obtain a value for \( \hat{\beta} \):

\[
\hat{\beta} = \frac{4\alpha A - 9bS}{6\alpha A - 9bA}
\]  

(4)

Proof. See Appendix.

Proposition 1 tells us that whenever the threat of hostile takeover is large Firm 1 prefers to sell to an non-competitor rather than trying to retain the ownership and face the probability of a competitor taking the firm. Given that \( \hat{\beta} \) is increasing in \( \alpha \) we see that the probability of take over must be higher in order for the firm to find it optimal to sell as the synergy between the firm and subsidiary increases.

4.4 Extension of the model

We now extend the model to analyze Firm 1’s initial decision to carve-out. We consider the possibility of not carving out (NCO) in first place hence not obtaining the funding needed to optimize the company’s operations or finance new projects. Because of this we can assume that the profit obtained in environment NCO is lower than the profit of retaining the subsidiary after the carve-out by an amount \( \lambda \). Therefore in this case the profit obtained by not carving out is \( \Pi^R - \lambda \).
Figure 2: Extended model

To further develop the model we also take into consideration that in the carve-out environment (CO) there is a possibility of not having a non-competitor who values the firm more than what Firm 1 does. We define as $\varphi$ the probability that there exists a non-competitor willing to acquire the subsidiary, and as $(1 - \varphi)$ the probability that the contrary happens. When $\varphi$ is equal to 1 we are in the game described in the previous section, while when $\varphi$ is equal to 0 Firm 1 can either sell to a competitor...
or try to retain the subsidiary and face the probability of takeover. In this last scenario, if Firm 1 decides to not sell, therefore it tries to retain the ownership over the subsidiary, the expected payoff will be $\beta \Pi^T + (1 - \beta)\Pi^R$. If instead, it decides to sell to the competitor the profit will be $\pi^T + S$ which as previously showed is less than both $\Pi^R$ and $\pi^S + S$ (See Figure 2). With these extensions the set of strategies for Firm 1 now is:

$$S_1 = \{(NCO); (CO, (S, S)); (CO, (S, DS)); (CO, (DS, S)); (CO, (DS, DS))\}$$

where, for example, strategy $(CO, (S, DS))$ represents the strategy where Firm 1 first carves out, and then sells if there is a non-competitor willing to buy, and does not sell if there is no non-competitor willing to buy. The optimal strategy $s_1^*$ now depends both on the threat of takeover and on the probability of having an indirect competitor willing to buy. By looking at the subgames after the carve-out we can identify the dominated strategies. If we are in the environment in which we don’t have a non-competitor to sell to, i.e. when $\alpha = 1$, we see that Don’t Sell (DS) is a dominant strategy, namely if Firm 1 plays S it will get $\pi^T + S$ which is strictly lower than $\beta(\pi^T + S) + (1 - \beta)(\Pi^R)$. This means that whenever we are in the case in which there is no non-competitor willing to buy the subsidiary, Firm 1 is better off by trying to retain the ownership of it after the carve-out. Differently, when there is a non-competitor who values the firm more than Firm 1, i.e. $\varphi = 1$, the optimal strategy depends on the threat of takeover. It follows that, if the threat of takeover is high, i.e. when $\beta > \hat{\beta}$, Firm 1 should sell after the carve-out happened, while if the threat of takeover is small, i.e. when $\beta < \hat{\beta}$ Firm 1 should try to retain the ownership. By backward induction now we can look at the decision of whether to carve-out or not in the first period, depending on $\beta$. In the case where $\beta$ is high, it is optimal to carve out whenever the utility Firm 1 gets from not carving out is lower than the expected value of Selling (S) when there’s a non-competitor and Don’t Sell (DS) when there isn’t one, that is:
If $\beta > \hat{\beta}$ and $u_1(\text{NCO}) < u_1(\text{CO}, (S, DS))$ than in equilibrium, carving out in the first stage is optimal.

In the case $\beta$ is smaller or equal to $\hat{\beta}$ carve out happens in equilibrium whenever the expected value of not selling ($DS$) when there is a non-competitor and not selling when there isn’t leads to an higher utility than the one from not carving out, namely:

If $\beta \leq \hat{\beta}$ and $u_1(\text{NCO}) < u_1(\text{CO}, (DS, DS))$

then again, Firm 1 should optimally carve-out in the first period. The following lemma states that the optimal decision of Firm 1 is independent of $\varphi$ whenever $\beta$ is not to large.

**Lemma 2.** There exists $\bar{\beta} \in (0, 1)$ such that for all $\beta < \bar{\beta}$

$$u_1(\text{CO}, (DS, DS)) > u_1(\text{NCO})$$

Namely,

$$\bar{\beta} := \frac{\lambda}{\Pi_R - \Pi_T}$$

**Proof.** See Appendix.

According to the variation of the parameters $\beta$ and $\varphi$ we can therefore summarize our results, defined as 3 regions as stated in the following proposition.

**Proposition 2.** Whenever the cost of not carving out, $\lambda$, is large, the Nash equilibrium strategies are characterized by the following three regions in the space of ($\alpha, \beta$).

1. If $\beta < \hat{\beta}$ then Firm 1 optimally carves out and never sells the firm; $(CO, (DS, DS))$.

2. If $\hat{\beta} < \beta < \bar{\beta}$ then Firm 1 optimally carves out and sells when there is a non-competitor willing to buy, or carves out and try to retain otherwise; $(CO, (S, DS))$. 

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(3) If \( \beta > \bar{\beta} \) then there exists an increasing function \( f : [0, 1] \to [0, 1] \) such that whenever \( \varphi < f(\beta) \) the firm never carves out. Otherwise, the firm optimally carves out and sells only when there is a non-competitor willing to buy; NCO if \( \varphi < f(\beta) \). Otherwise, \((CO, (S, DS))\) if \( \varphi > f(\beta) \).

The results of proposition 2 are illustrated in the following graph.

Figure 3: A simulation of the results for \( A = 100, \alpha = 15, b = 1, S = 500, \lambda = .2\Pi_R \)
5 Conclusion

In this paper we analyzed the optimal divestment strategies of a diversified corporation in the presence of market competition. We constructed a two-stage model in which the firm, first decides whether to carve-out the subsidiary or not, and then, in case of carve-out, chooses whether to completely sell-off the business unit or try to hold control over it. We included in the model the probability, \( \beta \), of the subsidiary to be taken over by a hostile raider if after the carve-out, Firm 1 tries to retain the ownership. Moreover, we accounted for the possibility that in the market there are no non-competitors evaluating the subsidiary more than Firm 1, willing to buy the carved-out branch. Impling that with probability \( 1 - \varphi \) the subsidiary is undervalued by the buyer. We first showed that in the baseline game, where Firm 1 already carved-out and there is a non-competitor evaluating the firm more, the optimal strategy will be to fully divest if and only if the threat of takeover is high. In the other subgame instead, where there is no non-competitor Firm 1 should always try to retain the ownership rather than selling the business unit to a competitor. These first findings give a different result to the analysis made by McKinsey(2001). As it is showed in our model, selling off a subsidiary after its carve-out makes sense only when the threat of takeover is high. McKinsey(2001) reports that there may be the possibility that a competitor takes over the business unit, proposing the example of Citicorp’s takeover of Ford’s financial services carve-out. What the 2001 report fails to show though, is that in the case there is no threat of takeover, one shouldn’t expect a rational manager to sell-off the subsidiary.

After showing the result for the subgame, we further extended the model to include the probability \( \varphi \) of having a non-competitor evaluating the subsidiary more than the divesting firm. We then, analyzed the initial decision of the firm to carve-out the business unit, based on both \( \varphi \) and \( \beta \). We showed that Firm 1 doesn’t carve-out when \( \beta \) is very high and there is no non-competitor. Otherwise in the first stage it always carves out. The results imply that Firm 1 decides to carve-out even when the threat of takeover is high if the probability of having a buyer that is a non-competitor is high. Whenever the threat of takeover is instead very low, it
is convenient for Firm 1 to try to retain the subsidiary for all the values of $\varphi$, the reasoning behind this is that if Firm 1 doesn’t risk to lose the company it can exploit full control over the business unit together with having already obtained the benefits from the partial IPO. Lastly we showed that when $\beta$ assumes a moderate value, that is between $\hat{\beta}$ and $\bar{\beta}$ it should always carve-out in the first stage.

In this paper we proposed a model which considers implications on strategic decisions of divestitures. Namely, we demonstrated that if the threat of takeover is low, a firm should always carve-out even if it can only sell-off to a competitor. On the contrary, when the threat of takeover increases above a certain value it is crucial to consider what is the market of potential buyers of the subsidiary. Therefore, when a firm is deciding to divest one of its subsidiaries is it necessary that it has a clear understanding both of how such a decision could lead the company to face a hostile bidder, together with the market composition of non-competitors willing to buy the subsidiary in case of a further full divestment.
References


6 Appendix

Proof of Cournot Equilibria Given the following demand function:

\[ p(q^1, q^2) = A - b(q^1_k + q^2_k) \]

(1) in the environment in which Firm 1 decides to sell \( \pi_1^S = \pi_2^S = q_i p_i - \alpha q_i \)
we want to maximize \( \pi_i^S(q_1, q_2) \).

\[ F.O.C \quad \frac{\partial \pi_1}{\partial q_1} = 0 \]
solving for \( q_1 \) and \( q_2 \) we obtain the best reaction functions \( q_1^S(q_2) \) and \( q_2^S(q_1) \). By substituting one into the other we obtain:

\[ q_1^S = q_2^S = \frac{(A - \alpha)}{3b} \]

Then, by plugging the quantities in the demand function we get:

\[ p^S = \frac{(A + 2\alpha)}{3} \]

Finally the profit of Firm 1 under cournot competition, in environment (S) is:

\[ \Pi^S = \frac{(A - \alpha)(A + 2\alpha)}{3b} - \alpha \frac{(A - \alpha)}{3} + S \]

That is:

\[ \Pi^S = \frac{(A - \alpha)^2}{9b} + S \]

(2) In the takeover environment the competitor enjoys lower costs, namely \( \hat{\alpha} \).
therefore \( \pi_1^T = q_1 p_1 - \alpha q_1 \) and \( \pi_2^T = q_2 p_2 - \hat{\alpha} q_2 \) (by normalization we set \( \hat{\alpha} = 0 \)).
from the F.O.C we obtain:

\[ q_1^T = \frac{(A - 2\alpha)}{3b} \quad q_2^T = \frac{(A + \alpha)}{3b} \]
by substituting the quantities in the demand function we obtain the price:

\[ p^T = \frac{(A + \alpha)}{3} \]

and then we compute the profit of Firm 1 in the takeover environment:

\[ \Pi^T = \frac{(A - \alpha)^2}{9b} + S \]

(3) to compute the cournot equilibria in the environment in which Firm 1 retains the subsidiary, we observe that now the profit functions are inverted, namely:

\[ \pi_1^R = q_1p_1 - \alpha q_1 \quad \text{and} \quad \pi_2^R = q_2p_2 - \alpha q_2 \]

therefore, by symmetry we obtain:

\[ q_1^R = \frac{(A + \alpha)}{3b} \quad \text{and} \quad q_2^R = \frac{(A - 2\alpha)}{3b} \]

with the same price as in (2). We can therefore compute the profit of Firm 1, i.e.:

\[ \Pi^R = \pi^R = \frac{(A + \alpha)^2}{9b} \]

\[ \square \]

**Proof of Lemma 1** To prove that \( \Pi^R > \Pi^T \), we consider \( S^T \leq \Pi^R - \pi^T \), as the most the hostile raider is willing to pay for the subsidiary. That is the difference between the profit it will get by acquiring the firm (which corresponds to \( \Pi^R \) of Firm 1) and that it obtain by not acquiring it (which corresponds to \( \pi^T \) of Firm1). Therefore we see that \( \pi^T + S^T \leq \Pi^R \) implying that \( \Pi^R > \Pi^T \). To prove \( \Pi^R > \Pi^S \) we solve the disequation, and we see when this holds, that is:

\[ \frac{(A + \alpha)^2}{9b} > \frac{(A - \alpha)^2}{9b} + S \]
which implies:
\[
\frac{A^2 + \alpha^2 + 2A\alpha}{9b} > \frac{A^2 + \alpha^2 - 2A\alpha}{9b} + S
\]
Rearranging we obtain:
\[
S > \frac{4\alpha A}{9b}
\]
whenever this holds, \( \Pi^R \) is greater than \( \Pi^S \). Which we assumed holds in the explanation of the model.

**Proof of Proposition 1** By setting the profits of selling to a non-competitor equal to the expected profits obtained in trying to retain, i.e:

\[
\Pi^S = \frac{(A - \alpha)^2}{9b} + S = \beta(\frac{(A - \alpha)^2 - 2\alpha A}{9b} + S) + (1 - \beta)(\frac{(A + \alpha)^2}{9b})
\]
and solving for \( \beta \) and manipulation with some algebra we obtain the value \( \hat{\beta} \):

\[
\hat{\beta} = \frac{4\alpha A - 9bS}{6\alpha A - 9bA}
\]

It follows that, since as previously demonstrated \( \Pi^R > \Pi^T \), as \( \beta \) increases the expected profit of not selling decreases. Therefore:

Whenever \( \beta > \hat{\beta} \) then \( \Pi^S > \beta \Pi^T + (1 - \beta) \Pi^R \)
and viceversa.

**Proof of Lemma 2** \( \bar{\beta} \) is the value for which Firm 1 is indifferent between carving out in the first stage and do not carve-out in the first stage, that is:

\[
\Pi^R - \lambda = \beta \Pi^T + (1 - \beta) \Pi^R
\]
solving for $\beta$ we obtain the value of $\tilde{\beta}$:

$$\tilde{\beta} := \frac{\lambda}{\Pi_R - \Pi_T}$$

This shows that:

Whenever $\beta < \tilde{\beta} < \tilde{\beta}$ then $\beta \Pi^T + (1 - \beta) \Pi^R > \Pi^R - \lambda$

$\square$

**Proof of Proposition 2**

(1) By proposition 1, if $\beta < \tilde{\beta}$ then $(CO, (DS, DS)) > (CO, (S, DS))$.

Further, from lemma 2, we see that if $\beta < \tilde{\beta}$ then $\beta < \beta$, therefore:

$$u_1(CO, (DS, DS)) > u_1(NCO)$$

(2) If $\beta > \tilde{\beta}$ then $u_1(CO, (S, DS)) > u_1(CO, (DS, DS))$ and if $\beta < \tilde{\beta}$ then $u_1(CO, (DS, DS)) > u_1(NCO)$, implying that:

$$u_1(CO, (S, DS)) > u_1(NCO)$$

(3) to analyze the optimal strategy when $\beta > \tilde{\beta}$, We set :

$$\varphi \Pi^S + (1 - \varphi)[\beta \Pi^T + (\beta) \Pi^R] = \Pi^R - \lambda$$

If $\varphi = 0$ then $(CO, (S, DS)) = u_1(NCO)$ iff. $\beta = \tilde{\beta}$. instead

If $\varphi = 1$ then $\Pi^S > \Pi^R - \lambda$ when $\lambda$ is large.

We then fix $\beta$ to a value $\tilde{\beta} > \tilde{\beta}$ and $\forall \tilde{\beta} \in \varphi$ s.t. :

$$\tilde{\varphi} \Pi^S + (1 - \tilde{\varphi})[\tilde{\beta} \Pi^T + (1 - \tilde{\beta}) \Pi^R] = \Pi^R - \lambda$$

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Further $\forall \varphi > \bar{\varphi}$:

$$\bar{\varphi}\Pi^S + (1 - \bar{\varphi})(\bar{\beta}\Pi^T + (1 - \bar{\beta})\Pi^R) > \Pi^R - \lambda$$

Therefore $\exists f(\beta)$ s.t $\varphi = f(\beta)$ implying that:

$$u_1(CO, (S, DS)) = u_1(NCO)$$

By continuity, if $\varphi > f(\beta)$:

$$u_1(CO, (S, DS)) > u_1(NCO)$$

$\square$